**Thinking about Knowledge**

1 知識の種類

（1）　人、場所、事物についての知識

（2）　How 型知識

（3）　What 型知識

「私は野田首相を知っている」、「和子は他の誰より東京を熟知している」

「この子は平泳ぎの名手で、どう泳ぐかよく知っている」

「理系の学生は解析学が何を主張している理論か知っている」

What 型の知識は命題で表現できる知識（*propositional knowledge*）である。

2 命題的な知識

*S* は *p*（であること）を知っている（*S* knows that *p*.）。

*S* の代表は人間であり、*p* は命題を指している。

では、「命題」とは何なのか。文や言明の主張している内容で、その内容は「意味」と呼ばれてきた。命題は真か偽のいずれかである。事実（fact）とは真なる命題である。命題は信念、欲求、後悔、希望、その他の命題的態度（propositional attitudes）の対象である。

3 分析

命題的知識の分析とは、命題的知識を定義すること、つまり、命題的知識の必要十分条件を明らかにすることである。

*S* knows that *p* iff […]

**The JTB Analysis of Knowledge**

JTB

*S* knows that *p* iff (1) *p* is true, (2) *S* believes that *p*, and (3) *S* is justified in believing *p*.

1 Condition (1): truth requirement

命題を知るにはその命題が真でなければならない。偽の命題は知ることができない。

「*S* は *p* を知っているが、その *p* は真ではない」

この文はどのような意味で奇妙なのか。

「*S* は自分が *p* を知っていると思っているが、その *p* は真ではない」

この文はどのような意味で奇妙でないのか。

命題が真であるとはどのようなことなのか。

対応説と整合説、真理述語の形式的な振舞い、内容の特徴

2 Condition (2): belief requirement

命題を信じることはそれが真であることを受け入れる、あるいは、それを真とみなすことである。

「*S* は *p* を知っているが、*p* だと信じていない」
この文はどのようない意味で奇妙なのか。

3 Condition (3): justification requirement
*S* が *p* を知っていれば、*S* は *p* への正当化された信念をもっていなければならない。では、正当化された信念とは何なのか。

A belief based on good evidence, or held for a good reason

*科学的な実験、観察による証拠の正当性、験証、検証、確証
データや情報の正当性は科学活動ではどのように保証されているだろうか。あるいは、裁判において証拠の正当性はどのように保証されるのか。

**The Gettier Problem**
JTB は次のことを含意する。
*p* の正当化された真なる信念をもつことは、*p* を知るのに必要である、つまり、誰でも *p* を知っているなら、*p* の正当化された真なる信念をもっている。
*p* の正当化された真なる信念をもつことは、*p* を知るのに十分である、つまり、誰でも *p* の正当化された真なる信念をもっているなら、*p* を知っている。
Gettier はこの JTB への反例をつくった。*p* の正当化された真なる信念をもつようにみえても、*p* を知っているようには思えない場合があるというのが、その反例の一つである。

知識の基礎付け主義 (Foundationalism)
（1）数学の諸例：ユークリッド幾何学、集合論、確率論、実数等

（問）小学校でいきなり無理数や実数を習っただろうか。では、なぜそれらを最初に教えないのか。単純にそれらが複雑だからだろうか。

（問）自然数から始めて、どのように段階的に整数、有理数、無理数、実数を構成していくかの概略を述べよ。

数の構成から見えてくる、様々な数の間の関係は、段階的、階層的、構成的、蓄積的、拡張的といった形容詞で表現される。数の構成順序に従って実数がもつ性質を基礎から説明する、基礎からつくり出すことは、実数自体が明晰・判明なものから構成的に定義されていることを示しており、その定義上確かな基礎をもつことが自ずとわかる。

***ブルバキの試み:Bourbaki’s Reforming Mathematics**
Bourbaki members all believed that they had to completely rethink mathematics. As explained by Dieudonné

“if the mathematics set forth by Bourbaki no longer correspond to the trends of the period, the work is useless and has to be redone, this is why we decided that all Bourbaki collaborators would retire at age 50.” Bourbaki wanted to create a work that would be an essential tool for all mathematicians. Their aim was to create something logically ordered, starting with a strong foundation and building continuously on it. The foundation that they chose was set theory which would be the first book in a series of 6 that they named “*éléments de mathématique*”(with the 's' dropped from mathématiques to represent their underlying belief in the unity of mathematics). Bourbaki felt that the old mathematical divisions were no longer valid comparing them to ancient zoological divisions. The ancient zoologist would classify animals based on some basic superficial similarities such as “all these animals live in the ocean”. Eventually they realized that more complexity was required to classify these animals. Past mathematicians had apparently made similar mistakes : “the order in which we (Bourbaki) arranged our subjects was decided according to a logical and rational scheme. If that does not agree with what was done previously, well, it means that what was done previously has to be thrown overboard.” After many heated discussions, Bourbaki eventually settled on the topics for “éléments de mathématique” they would be, in order:
I Set theory
II Algebra
III Topology
IV Functions of one real variable
V Topological vector spaces
VI Integration

（問）ユークリッド幾何学についても、そのギリシャ以来の公理系、さらにはヒルベルトの新しい公理系等と比較し、何が構成的な知識の姿を生み出しているか考えてみよ。（言語、定義、演繹的証明、定理、定理間の関係、解釈等の用語を使って述べてみよ。）

（2）物理学を基礎にした自然科学の階層的な知識

存在するものの間での階層性とそれを表現する理論の間での階層性は厳密に考えれば異なっている。存在のレベル、例えば原子レベルと天体レベルには異なる理論があり、それら理論は相互に無関係であるといった場合が多い。だが、従来存在レベルと知識レベルの間には階層性の類似した関係があると願望されてきたにもかかわらず、量子力学と相対性理論の関係に見られるように階層的な世界全体を総合的に表現する理論は未だにない。世界が階層構造をもつという考えは比較的新しいものだが、それを促した知識はまだそれを完全に表現するという目標を達成できないでいる。そのため、それは夢であるにもかかわらず、統一理論（unified theory）という呼び名で言及されている。

（問）原子論（atomic theory）は古くからある考えだが、存在の基礎づけにどのような役割を果たしてきたか（ギリシャ時代の原子論、化学革命で生み出された化学的な原子論、素粒子理論等での「原子」の役割をヒントにしてみよう。）

（問）知識レベルでの原子論的な知識観を論理システム、言語システムをもとに、理論の階層的な配置として描いてみよ。また、Wittgenstein の（Russell が命名した）論理的原子論（logical atomism）

はどのような知識論か調べてみよ。

「還元」という概念も階層と並んで存在や知識の相互関係を表現するのに用いられてきた。対象や理論をより単純でわかりやすい対象や理論に還元することによって、対象の存在や知識の正当性、信頼性を基礎づけるという仕方で使われてきた。特に、理論間の還元は科学者の科学的仕事の一つとして研究されてきた。

理論間の還元例：熱力学と統計力学*、様々な光学、古典力学と相対性理論、古典力学と量子力学
*下の英文を読み、統計力学と情報理論の関係も考えてみよう。

（問）歴史学、記述的科学、臨床的知識、記録、伝承、物語、博物学といった名称は構成的でない知識のあり方を共通に含んでいる。幾つかは説明理論（explanatory theory）ではなく、記述あるいは記載の理論（descriptive theory）とも呼ばれてきた。これらの名称を参考にして説明理論と記述理論の違いを多面的に述べてみよ。

（問）記述的な知識を基礎づけることは果たして可能なのか。可能ならどのような方法が考えられるか（知識の内容ではなく、その表現手段に注目してみよ）。

**Entropy in thermodynamics and information theory**

There are close parallels between the mathematical expressions for the thermodynamic entropy, usually denoted by *S*, of a physical system in the statistical thermodynamics established by Ludwig Boltzmann and J. Willard Gibbs in the 1870s; and the information-theoretic entropy, usually expressed as *H*, of Claude Shannon and Ralph Hartley developed in the 1940s. Shannon, although not initially aware of this similarity, commented in it upon publicizing information theory in *A Mathematical Theory of Communication*.

Equivalence of form of the defining expressions

**Discrete case**

Boltzmann's tombstone, featuring his equation $S = k \log W$

The defining expression for entropy in the theory of statistical mechanics established by Ludwig Boltzmann and J. Willard Gibbs in the 1870s, is of the form:

$$S = -k \sum_i p_i \log p_i$$

where $p_i$ is the probability of the microstate $i$ taken from an equilibrium ensemble.

The defining expression for entropy in the theory of information established by Claude E. Shannon in 1948 is of the form:

$$H = -\sum_i p_i \log p_i$$

where $p_i$ is the probability of the message $m_i$ taken from the message space $M$.

If all the microstates are equiprobable (a microcanonical ensemble), the statistical thermodynamic entropy reduces to the form on Boltzmann's tombstone,

$$S = k \log W$$

where $W$ is the number of microstates.

If all the messages are equiprobable, the information entropy reduces to the Hartley entropy

$$H = \log |M|$$

where $|M|$ is the cardinality of the message space $M$.

The logarithm in the thermodynamic definition is the natural logarithm. It can be shown that the Gibbs entropy formula, with the natural logarithm, reproduces all of the properties of the macroscopic classical thermodynamics of Clausius. (See article: Entropy (statistical views)).

The logarithm can also be taken to the natural base in the case of information entropy. This is equivalent to choosing to measure information in nats instead of the usual bits. In practice, information entropy is almost always calculated using base 2 logarithms, but this distinction amounts to nothing other than a change in units. One nat is about 1.44 bits.

The presence of Boltzmann's constant $k$ in the thermodynamic definitions is a historical accident, reflecting the conventional units of temperature. It is there to make sure that the statistical definition of thermodynamic entropy matches the classical entropy of Clausius, thermodynamically conjugate to temperature. For a simple compressible system that can only perform volume work, the first law of thermodynamics becomes

$$dE = pdV + TdS$$

But one can equally well write this equation in terms of what physicists and chemists sometimes call the 'reduced' or dimensionless entropy, $\sigma = S/k$, so that

$$dE = pdV + kTd\sigma$$

Just as $S$ is conjugate to $T$, so $\sigma$ is conjugate to $kT$ (the energy that is characteristic of $T$ on a molecular scale).

**Continuous case**

The most obvious extension of the Shannon entropy is the differential entropy,

$$H[f] = -\int_{-\infty}^{\infty} f(x) \log f(x) dx$$

But it turns out that this is *not* in general a good measure of uncertainty or information. For example, the differential entropy can be negative; also it is not invariant under continuous co-ordinate transformations.

More useful for the continuous case is the **relative entropy** of a distribution, defined as the Kullback-Leibler divergence from the distribution to a reference measure *m*(*x*),

$$D_{KL}(f(x) \,||\, m(x)) = \int f(x) \log \frac{f(x)}{m(x)} dx$$

(or sometimes the negative of this).

The relative entropy carries over directly from discrete to continuous distributions, and is invariant under co-ordinate reparametrizations. For an application of relative entropy in a quantum information theory setting.

(Brief history of information)

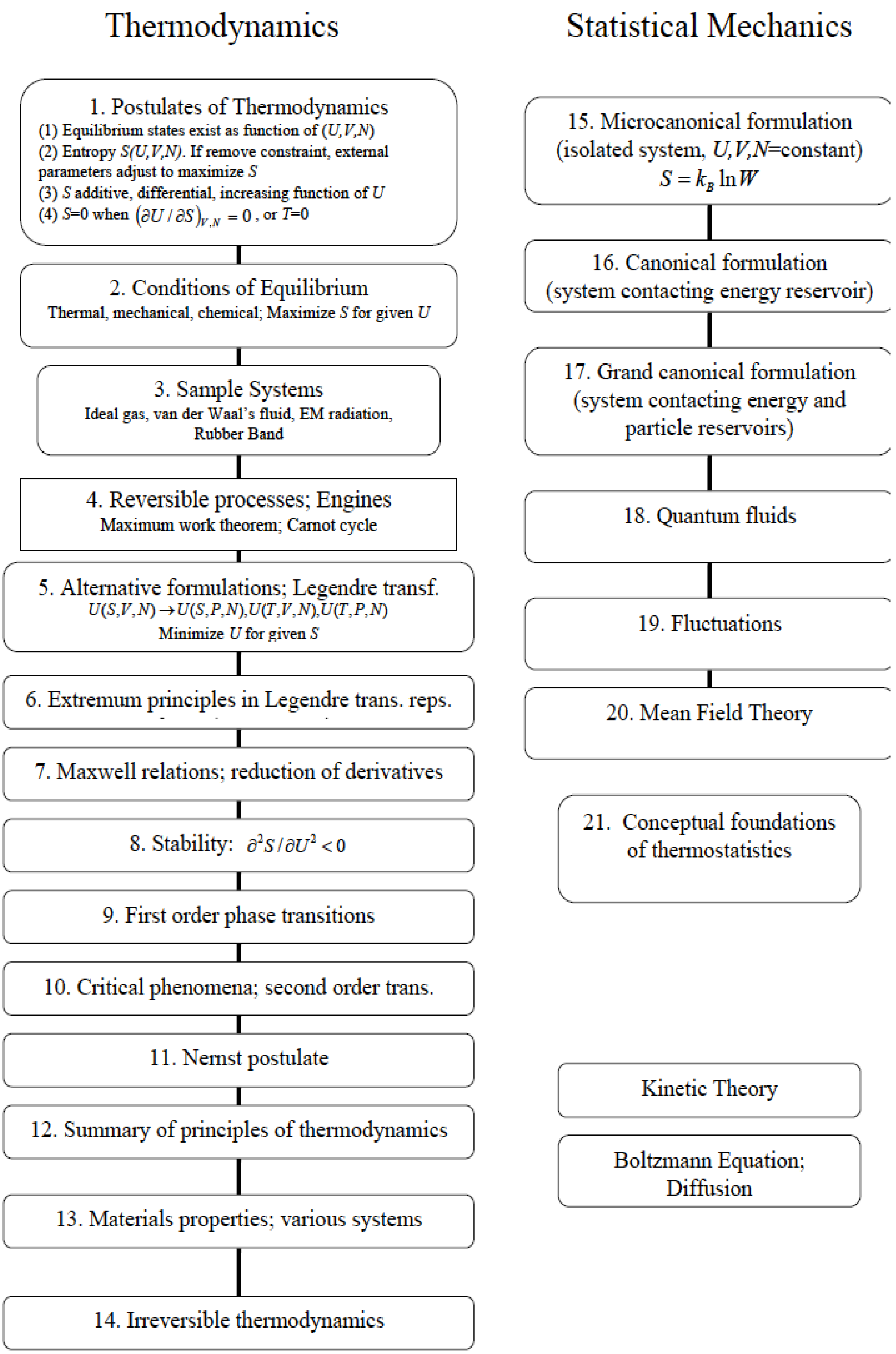
In less than two decades of the mid-twentieth century, the word *information* was transformed from a synonym for *knowledge* into a mathematical, physical, and biological quantity that can be measured and studied scientifically.

In 1939, Leo Szilard connected an increase in thermodynamic (Boltzmann) *entropy* with a*ny increase in information* that results from a measurement, solving the problem of "Maxwell's Demon," the thought experiment suggested by James Clerk Maxwell, in which a reduction in entropy is possible when an intelligent being interacts with a thermodynamic system..

In the early 1940s, digital computers were invented, by Alan Turing, Claude Shannon, John von Neumann, and others, that could run a stored program to manipulate stored data.

Then in the late 1940s, the problem of communicating digital data signals in the presence of noise was first explored by Shannon, who developed the modern *mathematical theory of the communication of information*. Norbert Wiener wrote in his 1948 famous book *Cybernetics* that "information is the negative of the quantity usually defined as entropy," and in 1949 Leon Brillouin coined the term "*negentropy*."

Finally, in the early 1950s, inheritable characteristics were shown by Francis Crick, James Watson, and George Gamow to be transmitted from generation to generation in a digital code.




付録

次の説明を読んで、イタリックの部分の論証を自分で再構成してみよ。

**Thomson's lamp**

Thomson's argument is roughly as follows: suppose there is a lamp which has a switch that, when pressed, turns the lamp on if it was previously off and off if it was previously on. Suppose, then, starting with the lamp off, that I press the switch an infinite number of times, pressing it once after a minute, again after a half-minute, again after a quarter-minute, and so on ad infinitum. But then, after two minutes have passed, is the lamp on or off? *Clearly it must be either on or off. But it cannot be on, because I never once turned it on without subsequently turning it back off. And it cannot be off, because I never once turned it off without subsequently turning it back on. So it can be neither on nor off. But if it must be either on or off and at the same time neither on nor off, then we have a contradiction. And so the supertask cannot have been performed.*

＊　上の論証を理解するための参考として以下の説明を挙げておこう。

The above is the argument Thomson gave and shows that the **supertask** is *logically impossible*. Let's look at the Thomson lamp scenario more closely. There is a lamp with a switch, taking the lamp from off to on and *vice versa*. For *n* even, the lamp is on; for *n* odd, the lamp is off. What state is the lamp in at 1, that is, after two minutes? The difficulty is that the state of the lamp does not converge to a single value as the time tends to 1. Rather, it alternates between two states faster and faster. So there is no limiting state of affairs which the lamp approaches as 1 draws nearer. It is problematic, therefore, to read off from our description what state the lamp will be in at 1.

Suppose, in describing the case, we stipulate:

L-1

The lamp cannot be on unless it is directly caused to be on by someone toggling the switch on, or by its initial manufacture,

where for a switch-toggling episode at $t$ to *directly cause* some state of the lamp at $t_0$, it must not be 'cancelled' by someone toggling the lamp back off again in the time between $t$ and $t_0$. In the context of L-1, it follows from the description of the Thompson lamp scenario that, all else equal, the lamp will be off at 12. Every switch-toggling that might directly cause the lamp to be on has been cancelled, in the relevant sense, and by L-1, the lamp can only be on if some uncancelled switch-toggling caused it to be on.

Suppose next we were to add:

L-2

The lamp cannot be off unless it were directly caused to be off by someone toggling the switch, or by its initial manufacture.

Given this, we could argue, in a way that exactly parallels the above, that the lamp must be on at 1. The conjunction of the two principles thus leads to the prediction that the lamp must be both on and off at 1 o'clock—a contradiction.